Delay and Fringe Rotation in Digital Beam Formers

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Abstract

An antenna is pointed at a source mechanically. An array of antennas must be pointed electrically. Electrical pointing involves adjusting the relative delays and phases of the signals coming from the antennas. The amount of delay error that can be tolerated for a given bandwidth is discussed from the point of view of a wideband system. Narrower bandwidths can tolerate greater delay error so a system that slices up the wide bandwidth into many narrow bandwidths is introduced as an option to solve the delay problem of a wide bandwidth system. If the cost per beam of providing a fine resolution delay exceeds the cost of frequency slicing then frequency slicing is recommended. The relative complexities of the two methods are compared.

Delay and Fringe Rotation

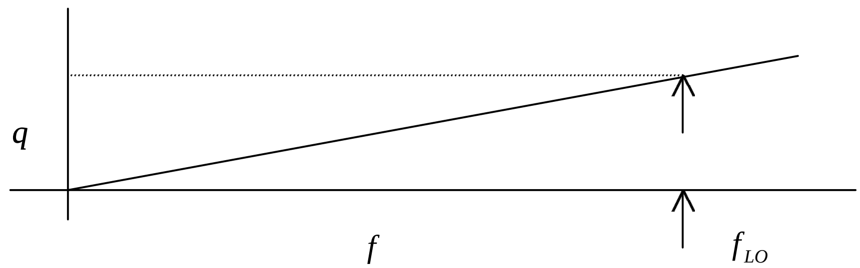
As the earth turns beneath a radio source in the sky, antennas in an array move at different speeds relative to each other. The motion of the tilting array causes the signal arriving at each of the antennas to experience a different Doppler shift. When the antenna signals are multiplied together the correlation or fringe is modulated by the Doppler shift. The array tilt also causes the arrival time for the signal at each of the antennas to be different. The Doppler shift produces fringe rotation or modulation and the delay causes loss of coherence. Analog systems have traditionally solved these problems by two devices. First, to make up for the delay difference, a switching delay line is introduced at each antenna, and second, the first LO at each antenna is tuned to a slightly different frequency to make up for the frequency shift in the signal caused by the antenna motion.

In a digital system the signal from each antenna is sampled at its Nyquist rate. The simplest way to delay a digital signal is to pass it through a shift register clocked at its sample rate[1]. Each stage of the shift register will delay the signal by one sample interval. The amount of delay is, then, equal to the number of shift register stages times the Nyquist interval. The delay can be changed by changing the number of shift register stages. Unfortunately, this delay change is “gritty” so that phase error is introduced into the measurement between delay jumps.

Consider the time-shifted Fourier transform pair.

$$h(t-t) \Leftrightarrow H(f)e^{-j2pft}$$

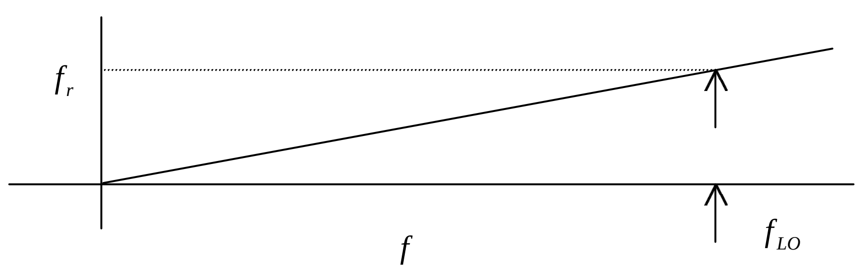
where phase angle $q = 2pft$ q in radians



Phase angle as a function of frequency through a delay.

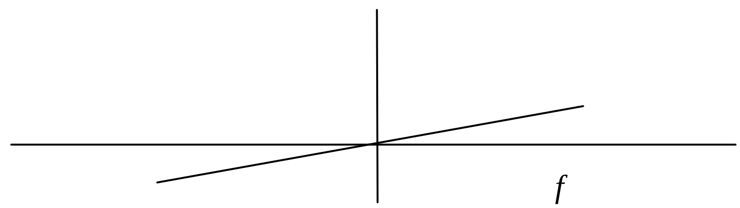
For a constant delay the phase angle remains constant at any given frequency. In our case the phase angle changes continually because of the change in delay.

$$\frac{dq}{dt} = f_r = 2pf\frac{dt}{dt}$$

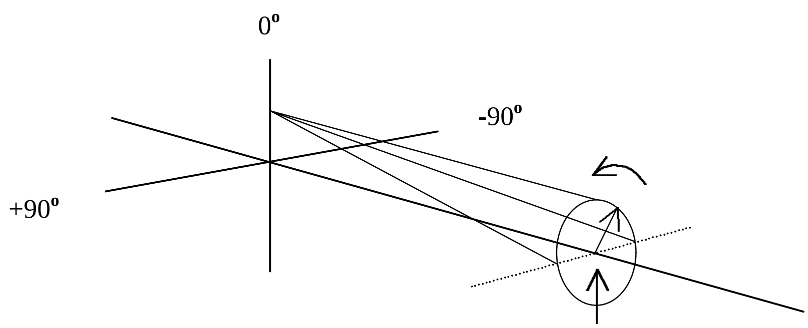


Fringe rate as a function of frequency through a constantly changing delay.

The fringe rate f_r may be removed by changing the LO to $f_{LO} + f_r$. Notice, however, that the fringe rate is removed at only one frequency and a fringe rate slope still exists across the passband.



Residual fringe rotation across the passband after the RF has been translated down to baseband.



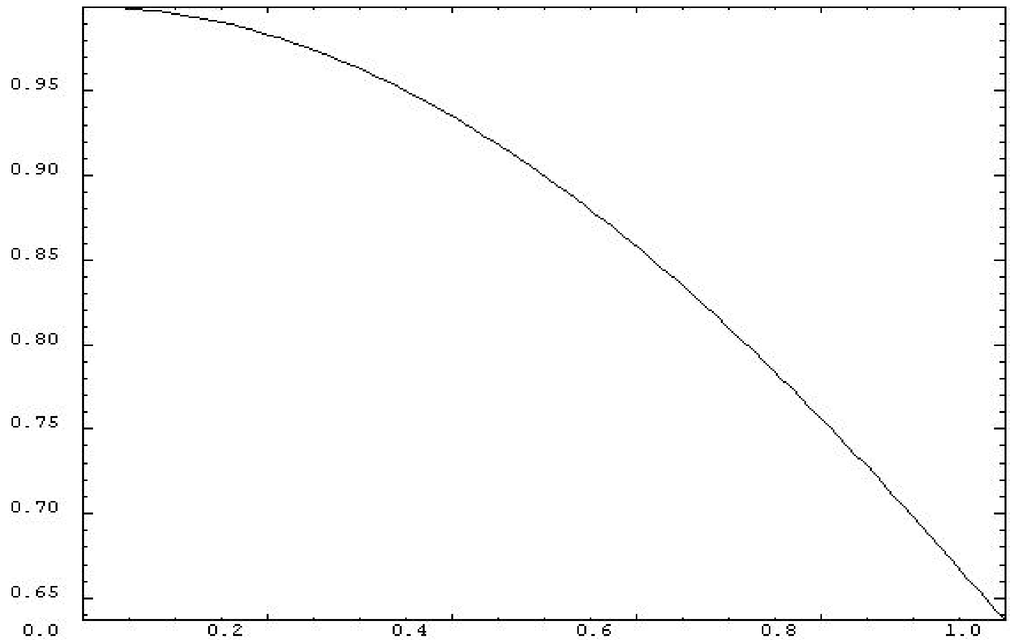
f_b

A 3D view of the rotating fringe at the edge of the baseband.

At the band edge f_b, the fringe rotates from –90 to +90 degrees as the delay error grows to the sample spacing. When an extra sample spacing worth of delay is introduced, the phase error at the band edge snaps back from +90 to –90 degrees. Since the signal does not add positively throughout the entire integration there is some signal loss, especially at the band edge where the motion is greatest. Since the instrument is designed to measure spectra it is necessary to determine the worst case error in the measurement at any frequency. This error occurs at the band edge.

$$signal = \frac{1}{a}\int_{-a/2}^{+a/2} \cos pxdx = \frac{2\sin(pa/2)}{pa}$$

Where a = a fraction of the Nyquist interval.



Plot of the integrated signal as a function of delay error relative to the Nyquist interval.

The above plot is derived from the phase activity of a signal relative to the phase center of the array. The phase activity from one antenna to another will be different. The plot shows that only 64% of the signal remains at the band edge with Nyquist spaced delay compensation. Since the noise remains the same it represents a loss in signal to noise ratio. This signal loss may not represent much of a problem except that the amount of loss in practice depends upon which antenna it’s crossed or compared with and where it’s pointed. The gain at the band edge becomes difficult to predict so power measurements at the band edge are inaccurate. It is necessary for finer delay adjustments to be made.

Delay Tracking

A simple way to provide finer delay resolution is to buy a more expensive A-D converter and over-sample the signal. The over-sampled signal can then be multiplexed into a memory that is configured as a fast shift register. Over-sampling by a factor of two can potentially recover 90% of the signal. Over-sampling by a factor of four can recover 97.4% and a factor of eight produces 99.3% signal recovery.

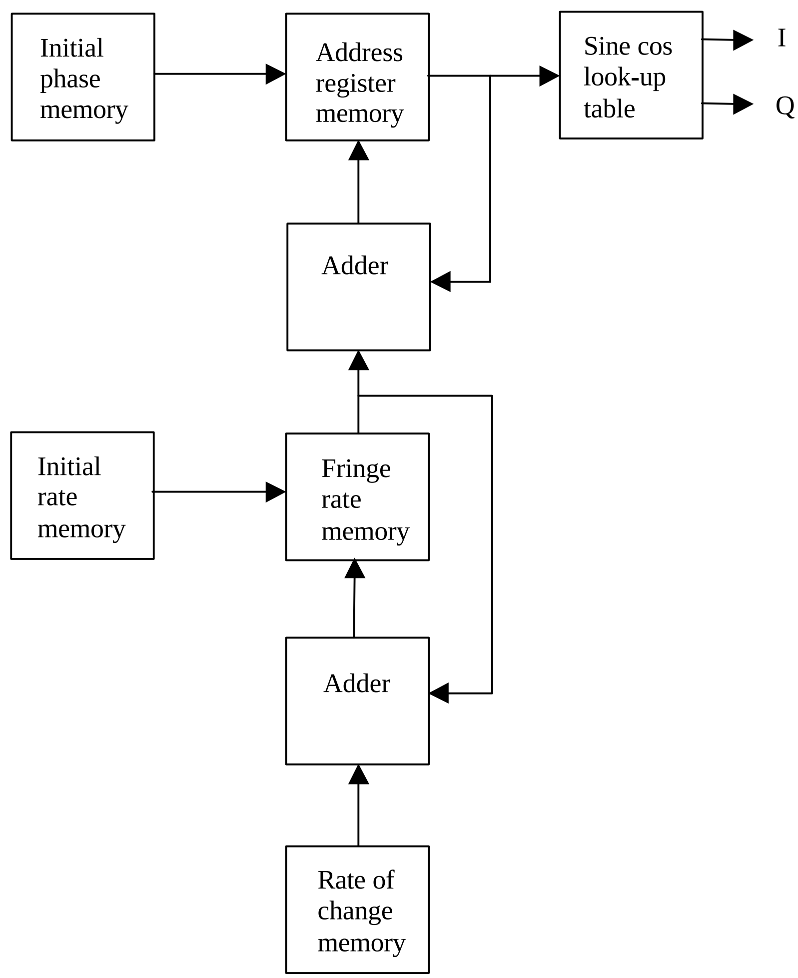
Some of the delay change grittiness can be removed by following a modest Nyquist sampling A-D converter with an interpolation filter. An interpolation filter determines the values of a signal between samples. The problem with an interpolation filter is the fact that it is made up of many coefficients that must be constantly changing. The interpolation filter may be combined with over-sampling to advantage. With over-sampling a more relaxed anti-aliasing filter may be used before the A-D converter. A polyphase interpolation filter may then be used to clean up the anti-aliasing characteristics as well as providing fine delay adjustment. Another approach may be to combine the interpolation filter with a windowing filter in front of an FFT machine to make a polyphase filter bank but at an increase in complexity.

A better place to apply fine delay tracking is after the polyphase filter bank. A complex phase rotator can multiply the polyphase filter output. A complex phase rotation at this point can accomplish two things. Not only can fine delay compensation be accomplished, but fringe rotation can be removed here rather than at the first LO. A filter bank is especially useful if you intend to break up the spectrum into frequency bands anyway. If as few as 8 channels of filter bank are used, then the delay precision for the narrower bandwidth is increased by a factor of 8 and 99.3% of the signal can be recovered. Loss of coherence, to be discussed later, requires a factor of 16 to recover 99.3% of the signal.

A 16-channel pipeline FFT is a rather simple device[2]. The rule of thumb for the relative complexity of an FFT device is that the number of complex multiply-adds required goes according to NlogN where N is the number of channels. If the logic being used clocks at the signals' Nyquist rate then only logN complex multiply-adds are required. It takes N clock pulses for the FFT to complete one conversion. The pipeline FFT consists of 16 stages of complex storage and four complex multiply-adders. The multiply-adders rotate the signal by 180, 90, 45 and 22.5 degrees. The first two of these rotations are trivial. By adding a multiplier-adder and 32 stages of complex storage, the input window to the FFT may be expanded by a pair of window widths. Six window widths should easily provide a good polyphase filter bank with steep sides and 60 db sidelobes. The cost of a 16-channel polyphase filter bank is five complex multiplier-adders and 128 stages of complex number storage. An interpolation filter would require at least one complex multiplier-adder per coefficient.

A Multichannel Quadrature Oscillator

It could be argued that following a filter bank with several quadrature mixers is a lot more expensive than having just one mixer in front of the bank. This is not the case. The data rate in front of the filter bank is the same as the rate following it. Only one mixer is required in both cases. The mixer following the filter bank must have a multiplexing capability.

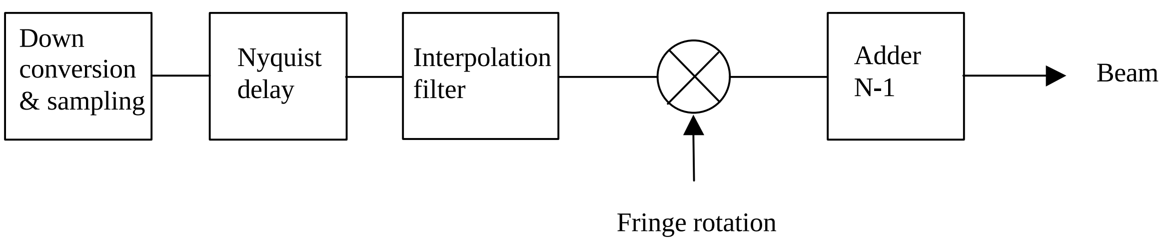


Example of a multichannel digitally controlled oscillator.

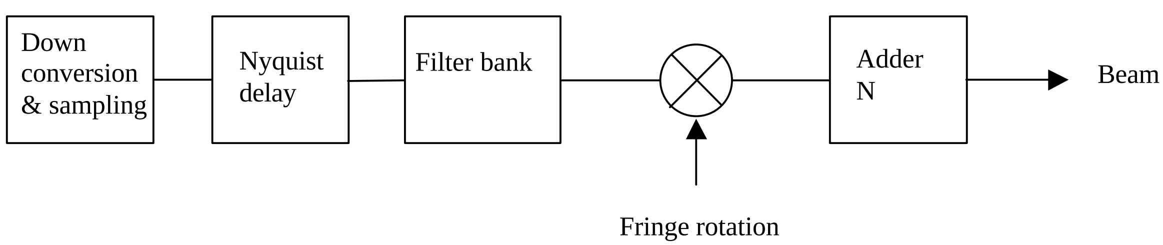
The above is a diagram of a digitally controlled oscillator that can multiplex between as many independent signals as there is capacity in the separate memories. At start-up the initial phase is loaded into the address register memory and the initial rate is loaded into the fringe rate memory. The fringe rate memory contains a number that determines the phase step per sample. At each sample interval the number in the fringe rate memory is added to the number in the address register memory and the address for the look-up table is stepped. The above digital oscillator has a provision for updating the fringe rate number so that a gradually changing fringe rotation can be tracked. The above oscillator can accommodate as many separate signals as can be stepped through between the sample intervals of the narrow-band filters.

Wide-band and Frequency Sliced Beam Formers

The multi-channel quadrature oscillator may follow the filter bank or it may follow a corner turner. If it follows the corner turner it would properly become part of the "back-end" where it could potentially be used to independently steer an electronic beam to anywhere in the telescope main beam.



Wide-band beam former



Frequency sliced beam former

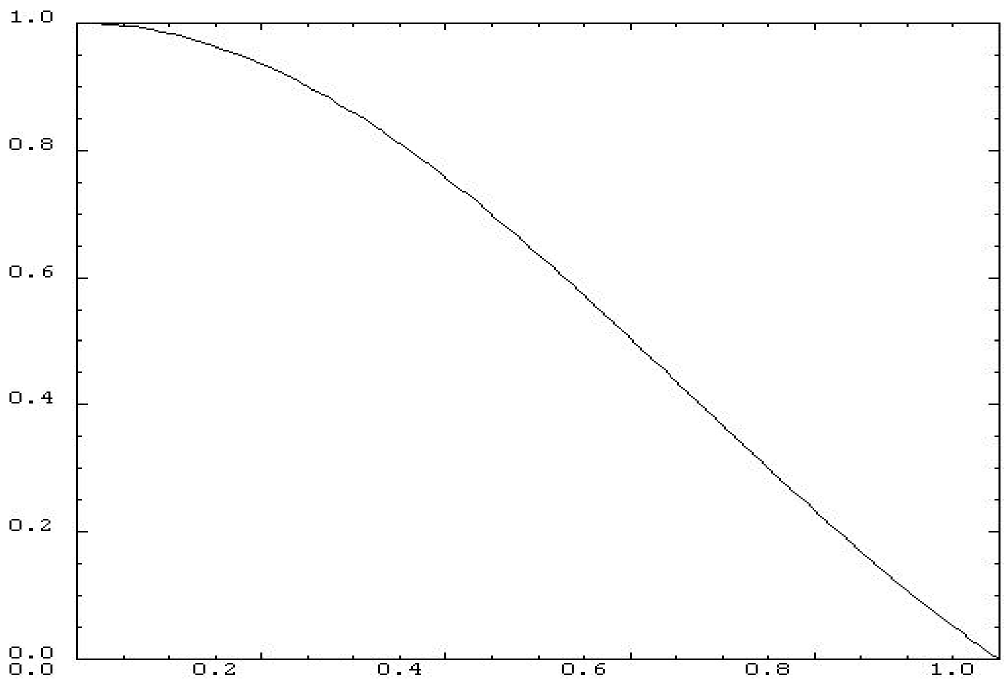
The number of frequency slices in the sliced beam former is M and the number of antennas is N. There are only N adders required in the sliced beam former rather than M (one for each slice) because the silicon used for the adders is as fast as the silicon used for the filter bank. If, for instance, there are three times as many slices as antennas then each adder has time enough to service three slices producing three beams.

The principal difference between the two systems is the interpolation filter and the filter bank. The interpolation filter could be less expensive than the filter bank leading to a conclusion that the wideband solution is less expensive. What must be considered, however, is how much does it cost to add another beam to the system? The wideband solution requires that another Nyquist delay line, interpolation filter, quadrature mixer and a set of N-1 adders be added for each new beam. Adding another beam to the frequency sliced solution requires only a digital multichannel quadrature mixer and a set of N adders.

Adding more beams by using separate multichannel quadrature mixers is very similar to following the filter bank with a discrete Fourier transform and the system begins to look like a direct imaging system. The bandwidth of each slice becomes important for consideration of both coherence and chromatic aberration. Unlike the previous consideration of phase twist across the band, delay error can cause a loss of signal across the whole band from a loss of coherence. While this signal loss represents a loss in signal to noise ratio and thus sensitivity, it is completely predictable. If you know what the delay error is then you know what the signal loss is. Chromatic aberration becomes a problem when spectral content at the extreme ends of the frequency slice come from different points in the sky.

The Coherence Problem

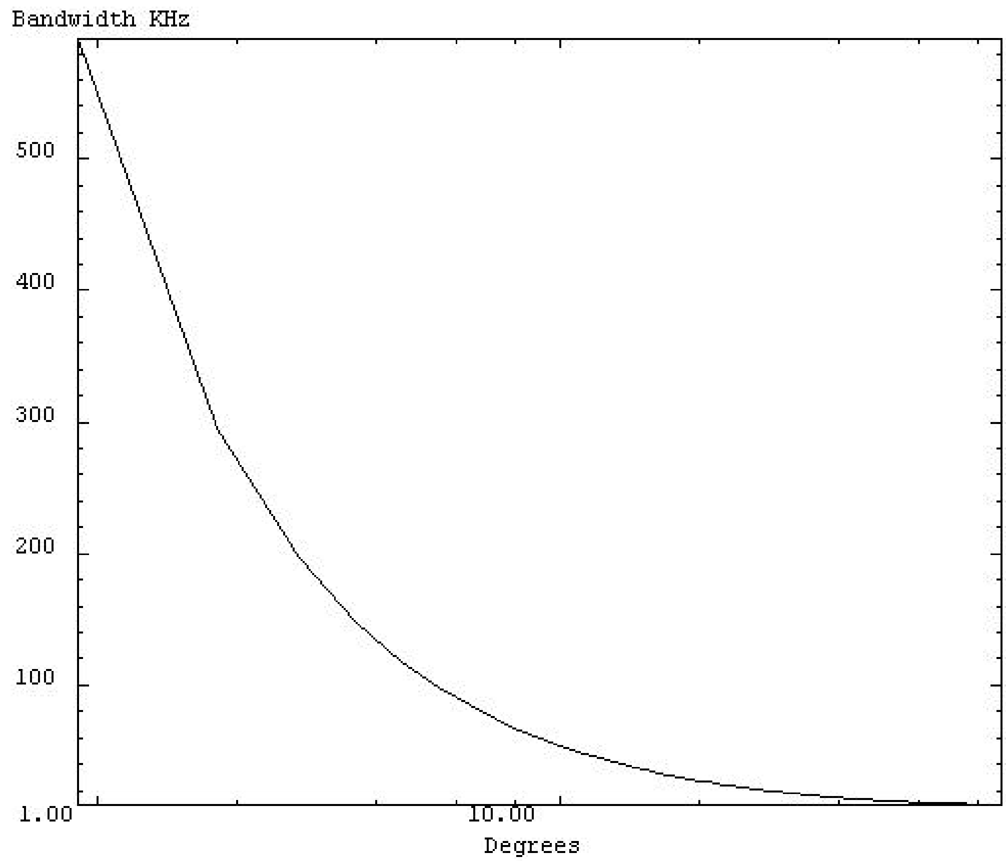
Following is a plot of the autocorrelation function for a square pass-band shape. The horizontal scale represents the fraction of the Nyquist spacing for the signal. As a rule of thumb, two samples are considered independent (they aren't really) if they are spaced one Nyquist interval apart corresponding to the first null of the sinc function. At spacings closer than the Nyquist interval the samples are correlated. The degree of



Plot of the sinc function.

correlation depends upon the shape of the pass-band but in general the pass-band can be considered square. Since a beam former must add all of the spectral components in the signal in phase, coherence is an important consideration. A delay error of one Nyquist interval at the array extremes will result in the complete cancellation of the signal from those antennas. If the delay error is confined to half of a Nyquist interval then 64% of the signal can be recovered. One fourth of a Nyquist interval recovers 90%, one eighth recovers 97.4% and one sixteenth recovers 99.3%. Signal loss from any antenna modifies its weighting in the aperture and could affect beam shape and side-lobe level. These considerations apply to both the wide-band and the frequency sliced beam formers and act in addition to the phase twist across the band discussed above.

A Nyquist delay line can be used at each antenna to electronically aim at the center of the main beam. The electronic beam can, then, be steered relative to the center of the main beam using multichannel quadrature mixers. If the multichannel quadrature oscillator described above steers the beam relative to the main beam in a frequency sliced beam former, then the bandwidth of each slice must be limited. At the extremes of the array the signal will have an extra path length to traverse equal to the sine of the beam angle times the array width. The signal coherence must be greater than this path length.



Plot of slice bandwidth vs. steering range in degrees from the main beam.

The above plot is for 99.3% signal recovery from a 1Km array. This plot is based upon the coherence of the signal. It shows that for a beam directed to 1.5 degrees off of the main beam, a bandwidth of less than 360 KHz is required. This degree of mobility is required to reach anywhere inside a three-degree main beam. A 100MHz beam would require at least a 278-channel filter-bank. A 400 MHz bandwidth requires at least 1112 channels.

A FFT doubles the number of channels with the addition of each complex multiply-add. Four additional complex multiply-adds would bring the number of channels from 16 to

256. Two more, beyond that, brings the number of channels up to 1024.

It could be argued that separating the electronic steering into two functions, delay and phase, adds to the complexity of the system. Unfortunately this complexity already exists. The mechanical and electronic pointing are already separate functions. The delay pointing could be lumped with the mechanical pointing since the delay would be confined to the center of the mechanical beam.

A system that completely eliminates the Nyquist delay could be constructed. All electronic beam steering could be accomplished using the multichannel quadrature mixers. A table can be constructed for an array with a maximum dimension of 1 Km and a signal transversal time of 3.3 usec. For the array pointed at the horizon the following signal loss may be expected at the extremes.

Nyquist	Bandwidth	Signal Loss
.5	75 KHz	36.4%
.25	37.5 KHz	10%
.125	18.75 KHz	2.6%
.0625	9.375 KHz	.7%

The above table depicts the amount of signal loss that can be expected at the outer boundaries of the array for a frequency sliced beam former with slices of the given bandwidths. A 100 MHz beam would require either an 8K or 16K filter-bank to get to the 1% or less signal loss level.

<u>The Chromatic Aberration Problem</u>

Returning to the time-shifted Fourier transform pair

$$h(t-t) \Leftrightarrow H(f)e^{-j2pft}$$

where phase angle $q = 2pft$ q in radians. A time delay can be replaced by a phase shift in a narrow-band system. In a wide-band system the beam is aimed electronically by adjusting individual delays from each antenna so that all of the signals add up constructively in one direction. In a narrow-band system, the same thing is done using phase-shifts rather than delays. The use of phase shifts is frequency dependent since the amount of phase-shift required to direct a beam of a given frequency in one direction, will direct a beam of another frequency in another direction. In a frequency sliced beam former, the bandwidth of the slices must be narrow enough to minimize the beam smearing that can occur. Beam smearing can produce a loss in sensitivity and a loss in resolution.

Considering a beam directed at an angle q from the center of the main beam. It can be shown that[3] for small angles

$$\frac{f_e}{f + f_e} q = q_e$$

where f is the frequency of the center of the frequency slice on the sky, f_e is the frequency at the band edge relative to the center frequency, and q_e is the error angle of the beam. For a beam directed at an angle of 1.5 degrees at a frequency of 1.4GHz the error angle for a 360KHz band-width frequency slice is .5″ of arc. The expected beam width for a naturally weighted 700m array is 75″ of arc[4]. The sensitivity loss due to beam smearing is then .67%. For a frequency of 11.2GHz the error angle is .04″ of arc. The expected beam width at that frequency is 9″ of arc. The sensitivity loss due to beam smearing is .4%.

[1] Romney, J.D. “Theory of Correlation in VLBI” ASP Conference Series, Vol 82, {\it Very Long Baseline Interferometry and the VLBA}, J.A. Zensus et al., editors; 1995.

[2] Groginsky, H.L. and Works, G.A. “A Pipeline Fast Fourier Transform” IEEE Trans. Computers, C-19(11), 1015-1019 (1970)

[3] Bos, A. “On Instrumental Effects in Spectral Line Synthesis Observations” Thesis de Rijksuniversiteit te Leiden June 11 1985.

[4] Wright M. “Astronomical Imaging with the ATA – III” ATA Memo 30.