ATA Memo #49

# Evaluating the TRW and ATA Feeds

David R. DeBoer

April 5, 2002

The *spillover efficiency* may be evaluated as

$$\varepsilon_{sp} = \frac{\Omega_{MB}}{\Omega_A}$$

where the beam solid angle is

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\varphi)\sin\theta d\theta d\varphi$$

and the main beam solid angle is

$$\Omega_{MB} = \int_0^{2\pi}\int_0^{\theta_{MB}} P_n(\theta,\varphi)\sin\theta d\theta d\varphi$$

where $\theta_{MB}$ is the main-beam half-angle. As there is incomplete angular data, define a "data" solid angle (assuming that at least the main beam has been measured) as

$$\Omega_D = \int_0^{2\pi}\int_0^{\theta_D} P_n(\theta,\varphi)\sin\theta d\theta d\varphi$$

where data exist out to $\theta_D$. The beam solid angle is then $\Omega_A = \Omega_D(1+\delta_D)$ where

$$\delta_D = \frac{\Omega_A - \Omega_D}{\Omega_D}.$$

$\delta_D$ should be a reasonably small quantity for which reasonable bounds may be postulated. If the directivity ($D$) is known, then $\delta_D = 4\pi/(\Omega_D D) - 1$.

The *taper efficiency* is defined for a symmetric prime-focus reflector as

$$\varepsilon_t = \frac{\cot^2(\theta_{MB}/2)}{\pi\Omega_{MB}}\left|\int_0^{2\pi}\int_0^{\theta_{MB}} \sqrt{P_n(\theta,\varphi)}\tan(\theta/2)d\theta d\varphi\right|^2.$$

Note that this assumes constant phase across the aperture. A *phase efficiency* may be defined to account for variations of phase. However, there currently exists no phase data for the TRW feed. A *polarization efficiency* may also be defined to take imperfect cross-polarization into account. Again, no cross-pol data currently exists for the TRW feed.

The *aperture efficiency* is then the product of the spillover, taper, phase and polarization efficiencies

$$\varepsilon_{ap} = \varepsilon_{sp}\varepsilon_t\varepsilon_{ph}\varepsilon_{pol}$$

The desired optimization is for $A_e/T_{sys}$, so the quantity to be maximized should be

$$\eta = \frac{\varepsilon_{ap}(\theta_{MB})}{1 + T_{sp}(\theta_{MB})/T_{A'}}$$

where $T_{A'}$ is the contribution of everything but spillover to $T_{sys}$ and $T_{sp}(\theta_{MB})$ is the contribution to $T_{sys}$ due to spillover. Then $A_e/T_{sys} = \eta A_p/T_{A'}$. For the ATA and the chosen illumination angle, the ratio $T_{sp}/T_{A'}$ is about 7% so as a reasonable first order model, one may express $T_{sp}(\theta_{MB})=(1-\varepsilon_{sp})T_{eff}$, where $T_{eff}$ ~ 16 K for the nominal ATA values. Therefore,

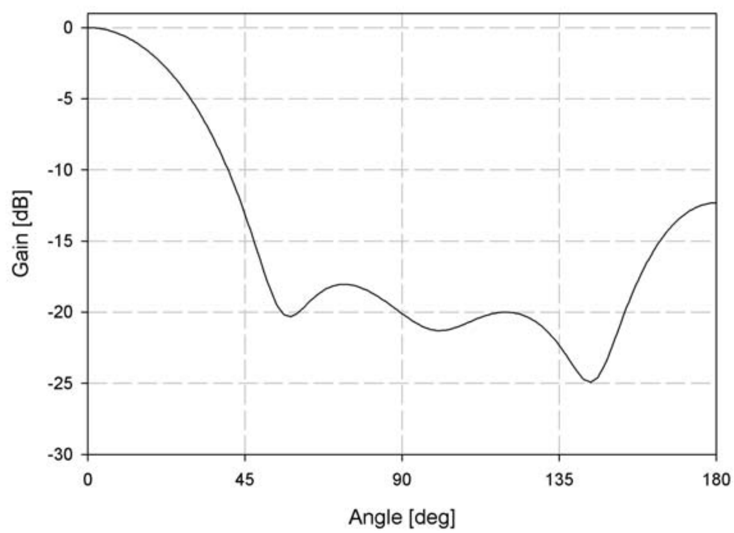
$$\eta \approx \frac{\varepsilon_{ap}}{1+0.4(1-\varepsilon_{sp})}$$

The analysis below will show both $\varepsilon_{ap}$ and $\eta$ for both the ATA and TRW feeds, where it will also be assumed for comparison that $\varepsilon_{ph}$ and $\varepsilon_{pol}$ are both equal to unity.

## The ATA Feed

The ATA feed is a pyramidal log-periodic feed most recently discussed in ATA Memo 45. The azimuthally averaged beam pattern is shown in Figure 1. The beam pattern is essentially frequency independent over a ~22:1 bandwidth.

Figure 1: Azimuthally-averaged ATA pattern




The integrated pattern extending from 140°-180° is about $\delta_D$=3% (used as a reference for the TRW analysis) and Table I lists several other parameters of the feed.

**Table I: Properties of the ATA feed.**

| | |
|---|---|
| Beam solid angle | 0.770 rad$^2$ |
| Directivity | 12.1 dBi |
| For mainbeam angle | 42° |
| Main beam solid angle | 0.630 rad$^2$ |
| Edge taper | -11.1 dB |
| $\varepsilon_{sp}$ | 82% |
| $\varepsilon_t$ | 87% |
| $\varepsilon_{ap}$ | 71% |
| $\eta$ | 0.66 |
| %beam above +10 dBi | 5.9% |
| %beam above +0 dBi | 27.8% |

## The TRW Feed

Figure 2a TRW Feed at 2 GHz

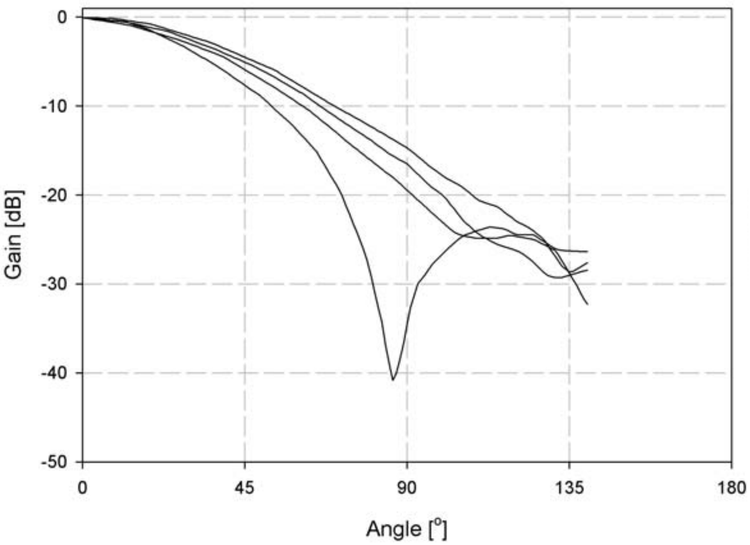



Figure 2b TRW Feed at 3 GHz

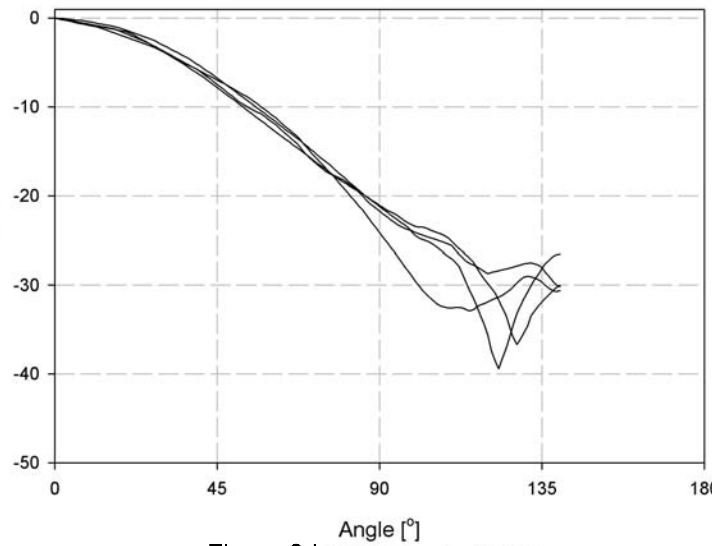



Figure 2c TRW Feed at 4 GHz

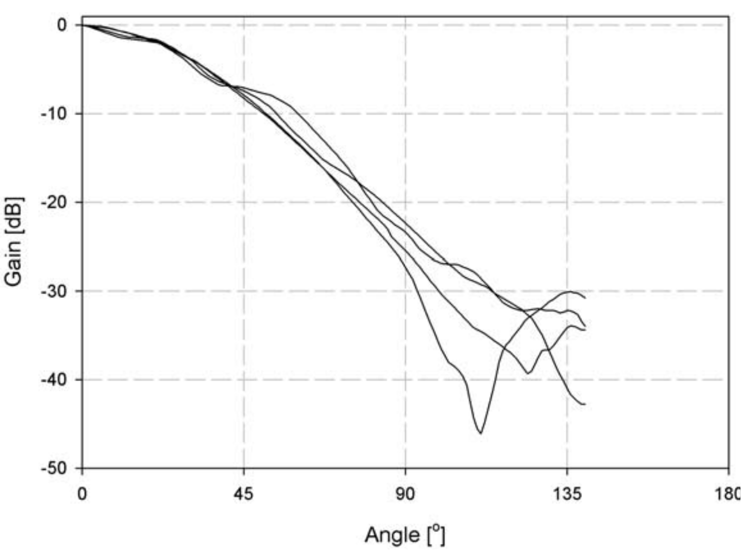
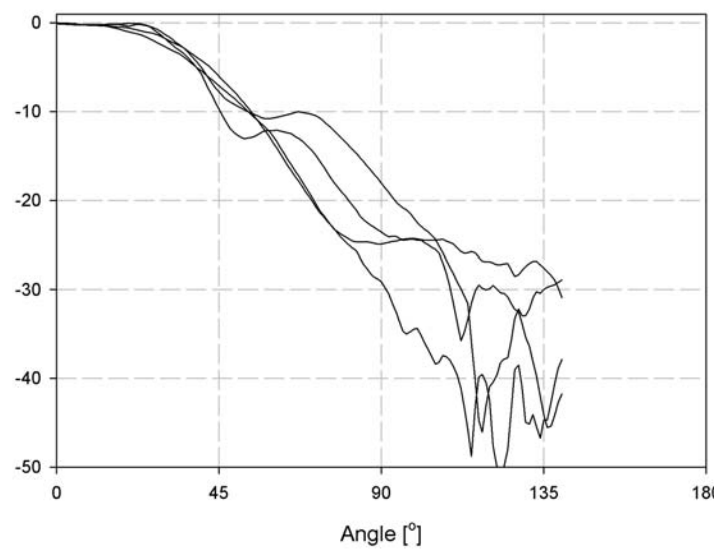



Figure 2d TRW Feed at 8 GHz




The analysis of the TRW feed is based on faxed normalized amplitude patterns extending from -140° – 140° at 2, 3, 4, 6, 8 GHz. Data for 2, 3, 4 and 8 have been read from the plots and are shown in Figure 2 above.

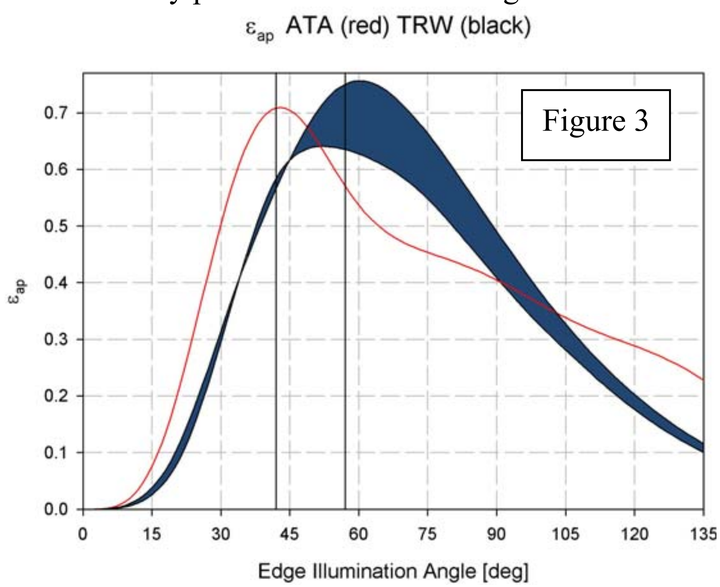
Since the patterns are incomplete, some integrated power must be assumed in the backward direction. Two values will be assumed: $\delta_D$=3% and 15%. The first value is the ATA feed power beyond 140°. The existing data leaves out about 12% of the total solid angle, so the second value of 15% seems a conservative upper limit. Table II lists some other parameters of the TRW feed, with the 2 values per frequency corresponding to the two values of $\delta_D$. Note that the 57° main beam half-angle is used since that is the widest angle for which the cross-pol nulling condition exists for our primary.

**Table II: Properties of the TRW feed**

| | 2 GHz | 3 GHz | 4 GHz | 8 GHz |
|---|---|---|---|---|
| Beam solid angle (rad$^2$) | 1.481 / 1.653 | 1.126 / 1.259 | 1.028 / 1.150 | 1.390 / 1.552 |
| Directivity (dBi) | 9.3 / 8.8 | 10.5 / 10.0 | 10.9 / 10.4 | 9.6 / 9.1 |
| For mainbeam angle deg (°) | 57 | 57 | 57 | 57 |
| Main beam solid angle (rad$^2$) | 1.218 | 0.984 | 0.915 | 1.237 |
| Edge taper (dB) | -11.6 | -9.9 | -11.3 | -10.7 |
| $\varepsilon_{sp}$ (%) | 82 / 74 | 87 / 78 | 89 / 80 | 89 / 80 |
| $\varepsilon_t$ (%) | 89 | 86 | 85 | 80 |
| $\varepsilon_{ap}$ (%) | 74 / 66 | 75 / 67 | 75 / 67 | 71 / 64 |
| $\eta$ | .69 / .60 | 0.71 / 0.61 | 0.72 / 62 | 68 / 59 |
| %beam above +10 dBi | 0 | 0.9 / 0.00 | 1.6 / 0.6 | 0 |
| %beam above +0 dBi | 50.3 / 47.7 | 44.5 / 42.3 | 44.9 / 42.4 | 36.2 / 34.8 |

## Comparison

The comparison is a bit problematic due to the uncertainty of the value of $\delta_D$ for the TRW feed. For a value of $\delta_D$ =10%, the two feeds have almost identical performance. Figure 3 shows $\varepsilon_{ap}$ and Figure 4 shows $\eta$ as a function of $\theta_{MB}$ for the two feeds, where the 3% value at 4 GHz and 15% value at 8 GHz are used for the limits for the TRW feed, since they provide the broadest range. The two vertical lines are at 42° and 57°.



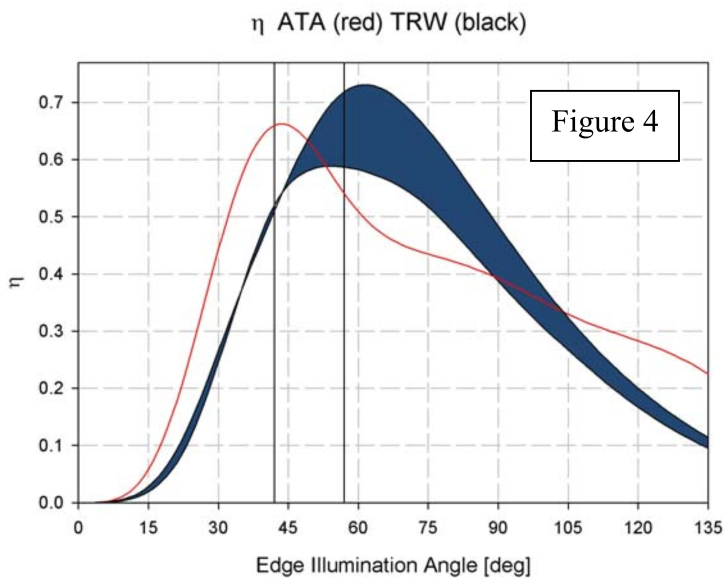
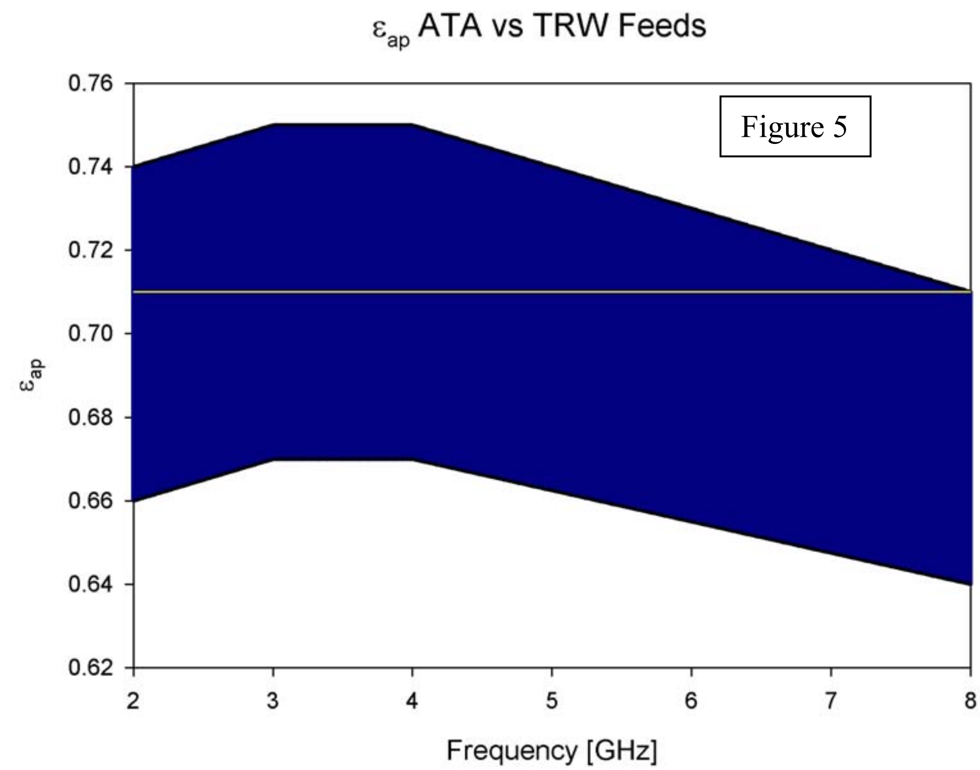
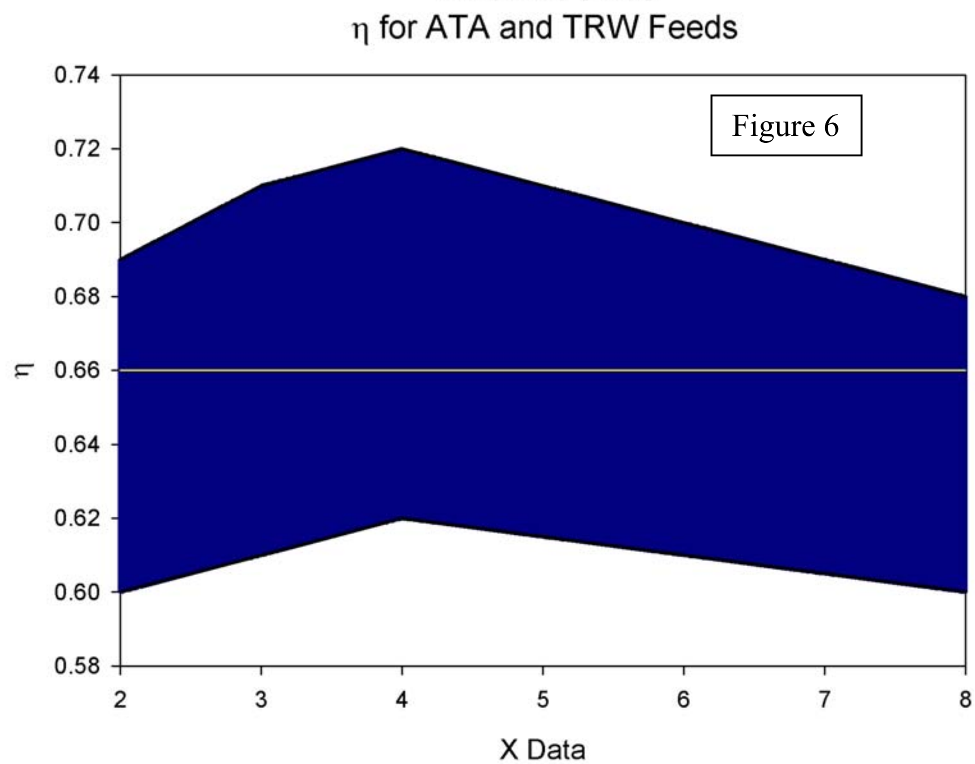




Figure 5 and 6 show the aperture efficiency and $\eta$ (again, with assumed perfect phase and cross-pol efficiencies), assuming the 42° and 57° half angles for the ATA and TRW feeds respectively. The blue range is the 3% and 15 % span and the yellow line is the ATA feed.




**Frequency [GHz]**

In terms of implementing either feed in the ATA configuration, the primary antenna remains the same and the subreflector changes to accommodate the different design. Figure 7 shows the two different geometries, with the TRW arrangement in red and the ATA feed in black. Table III lists the values

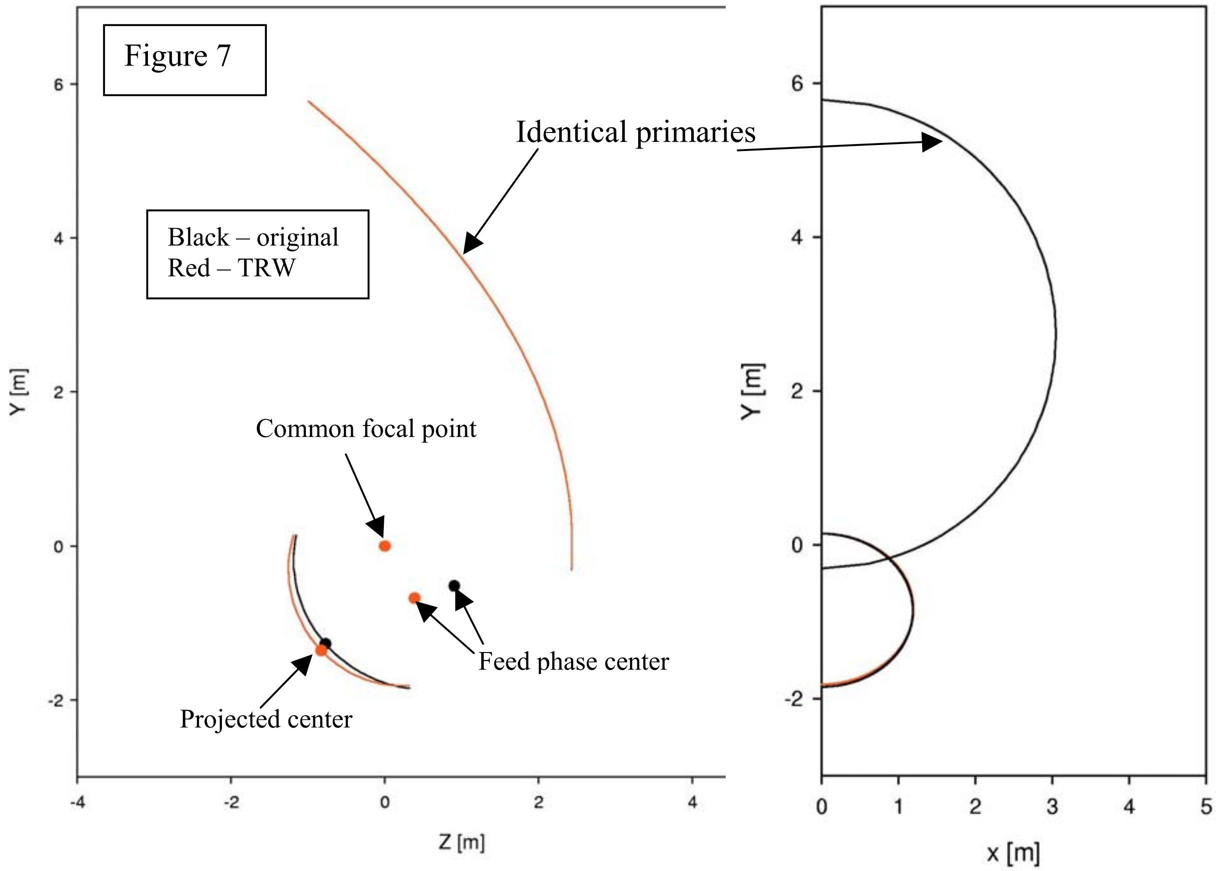



**Table III: Offset Gregorian parameters for the two feeds**

| *Reading parameters from greg42.des:* | *Reading parameters from greg57.des:* |
|---|---|
| *primary aperture diameter (D) = 6.090* | *primary aperture diameter (D) = 6.090* |
| *primary focal distance (f) = 2.436 (f/D=0.400)* | *primary focal distance (f) = 2.436 (f/D=0.400)* |
| *height of midpoint aperture (yc) = 2.740 (yc/D=0.450)* | *height of midpoint aperture (yc) = 2.740 (yc/D=0.450)* |
| *secondary subtended angle (ThetaH) = 42.000* | *secondary subtended angle (ThetaH) = 57.000* |
| *target secondary size = 2.400* | *target secondary size = 2.400* |
| For the ellipsoid: | |
| 2c = 1.0430 | 2c = 0.7800 |
| 2a = 3.3245 | 2a = 2.9776 |
| 2b = 3.1567 | 2b = 2.8736 |
| vertex = 2.1838 | vertex = 1.8788 |
| xi = 0.3071 | xi = 0.4344 |
| zeta = 1.1250 | zeta = 1.1250 |
| e = 0.3137 | e = 0.2620 |
| beta = 29.7630 | beta = 60.3100 |
| alpha = 53.9243 | alpha = 89.6188 |
| equivalent f/D = 0.6513 | equivalent f/D = 0.4604 |
| Sub-reflector size: | |
| T-B size in primary coordinates: 2.47531 | T-B size in primary coordinates: 2.47119 |
| L-R size in primary coordinates: 2.37106 | L-R size in primary coordinates: 2.38649 |
| Geometrical mean size: 2.399 | Geometrical mean size: 2.399 |
| Height: 1.990 | Height: 1.962 |
| Depth: 0.541 | Depth: 0.637 |
| | |

| | |
|---|---|
| Focal points: | |
| F1 is at (y1,z1): -0.518 0.905 | F1 is at (y1,z1): -0.678 0.386 |
| F0 is at (y0,z0): -0.843 0.614 | F0 is at (y0,z0): -0.780 0.005 |
| The center of the subreflector is at: | |
| (y0,z0) 0.000 1.838 | (y0,z0) 0.000 1.389 |
| (y1,z1) -1.270 -0.772 | (y1,z1) -1.358 -0.825 |
| Profile of main offset paraboloid with center at y1 = 2.740 | |
| Top point (5.7853,-0.9991) Bottom point (-0.3045,2.4264) | Top point (5.7853,-0.9991) Bottom point (-0.3045,2.4264) |
| Angle of face = -29.3578 | Angle of face = -29.3578 |
| Depth: 0.8294 m | Depth: 0.8294 m |
| Primary top-to-bottom distance = 6.987117 | Primary top-to-bottom distance = 6.987117 |

## Conclusions

The TRW feed does look quite promising; however we lack enough information to fully characterize it. The data we have spans only 2 octaves, the feed for the ATA is required to work over a 22:1 bandwidth. The ground plane portion of the TRW feed may not be made arbitrarily large to accommodate lower frequencies, since at some point it will occlude the beam waist of the Gregorian system, however it appears that sizes up to about 60-80 cm may be accommodated with the existing design.

Efficiency, impedance match, ohmic loss and cross-polarization all must maintain good performance across 4.5 octaves simultaneously. To fully characterize this feed, we therefore need to have these parameters fully evaluated over the 0.5 – 11.2 GHz ATA frequency range, as well as drawings of the feed to evaluate integration with the receiver and cryogenic package.