Notes on Configurations for the ATA

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ABSTRACT

Douglas recently presented a memo that summarizes RAL configuration science requirements for the ATA. Working within the guidelines that he has already presented, I have explored some choices for configurations that are based on Reuleaux triangles. Given 10 random filled Reuleaux distributions using identical constraints, the output sidelobe levels of the beam varies by nearly a factor of three, and the sidelobe distributions are very different. I examine the effects of masking out antennas due to the overall site perimeter, the labs and road, and the trees; the masking from the trees is the dominant contributor to poor sidelobe structure. As expected, there is a gradual tradeoff between an increase in beamsize and a decrease in the sidelobe levels that can be achieved given the site constraints. For an 86″ beam, one can achieve near-in sidelobes of 3.4% including masking, even without optimizing for sidelobe levels. For any choice of configuration, Reuleaux or otherwise, we should be able to achieve significantly better sidelobes than random configurations yield if we run an optimization program like Kogan's.

1. Introduction

The RAL science meetings on ATA configurations have generally yielded consensus that the configuration should be a compromise between good resolution on the one hand, and decent sidelobe levels and distributions on the other. Douglas and Mel have shown that we can achieve ∼69″ naturally-weighted beams with near-in sidelobes of ∼6.8% by choosing a random distribution of antennas distributed throughout the "lower triangle" region of the site at Hat Creek. The near-in sidelobes tend to be distributed in a somewhat complex pattern around the central lobe.

In the following, I explore configurations that are confined to within the outline of a Reuleaux triangle. A Reuleaux triangle is an curvilinear, equilateral triangle with arcs that have a curvature defined by the radius that is the distance between two of the vertices. Given the triangular geometry of the site at Hat Creek, a Reuleaux triangle probably gives the longest possible baselines for any of the regularly-shaped N-sided polygons which have constant diameter (including a circle).

As Jack and others have pointed out, the far-out sidelobe levels of the synthesized beam of an interferometer fall off as N^{-1} for large numbers of elements N, in a way that is more or less independent of the choice of antenna configuration. This means that if one wants to characterize a configuration by its sidelobe levels, then it is sufficient to first order to examine the near-in sidelobes.

2. Random Gaussian Distributions

First, I generated 10 random gaussian distributions of ~1000 points with a FWHM of 2 in arbitrary units. I clipped the distributions so that the points all fell within a Reuleaux triangle where the distance between two vertices was 2, rotated the triangle 15° in the clockwise sense to best fit the site geometry, and scaled the distributions to the size of the site. I then selected antennas from each distribution that have minimum separations from all other antennas of 11m. In this first experiment, I did not eliminate antennas that fell on "disallowed" regions like the trees, the labs and road, or that fell just outside the allowed site perimeter. The resulting configurations and their corresponding naturally-weighted beams (~84″) are shown in Figure 1 and 2; the beam sizes and number of antennas per configuration are listed in Table 1.

There is a surprising variety in the synthesized beams that result from these random distributions, especially given the large number of antennas for each of the configurations. The near-in sidelobes range from a maximum of 0.8% for the best case (case 4) to nearly a factor of three worse (2.2%) for some of the other cases (cases 3 and 8). Also, the distribution of sidelobes is considerably worse for some cases (e.g. cases 2, 10) than for others (case 4). Already we can conclude that, even for an idealized case with no masking for topography or local constraints, randomly generated configurations using ~350 antennas do not yield equally good choices of synthesized beams.

3. Random Uniform Distributions

I repeated the above experiment with 10 random distributions that were drawn from a uniformly distributed sample within a Reuleaux outline, rather than from a gaussian sample. The configurations and their synthesized beams are shown in Figures 3 and 4, and the beam sizes and number of antennas are listed in Table 2.

Again, there is a large spread in the output beams. The naturally weighted beams are somewhat smaller (~79″) than for the particular choice of the gaussian distributions above, and the near-in sidelobes are correspondingly worse, from ~2.7% for case 9, to 5.7% for cases 1 and 8. As shown in §5 below, a gaussian distribution that yields a similar beam size as that of case 9 has smaller (1.7%) near-in sidelobes. This suggests that it would be advantageous to choose a configuration that includes some tapering of the antenna distribution.

4. Effect of Masking for Site Geometry, Labs/Road, and Trees

Next, I explore the effects of masking antennas for the three following "disallowed" regions: (1) the overall site perimeter, (2) the lab buildings and road, and (3) the grove of trees; then I look at the composite effect of using all these masks together. For this experiment, I used case 4 in Table 1 and in Figures 1 and 2, since case 4 had the lowest and simplest sidelobe structure of any of the random configurations as well as a very round beam.

Figure 5 shows the results of the masking. In each case, I simply threw away antennas which were in the disallowed regions; I did not attempt to move them, even slightly, to allowed regions. One could clearly do better than this worst-case scenario by repositioning the masked antennas.

As Figure 5 shows, throwing away antennas that were outside the site outline makes only an incremental difference in the resulting synthesized beam: the near-in sidelobe levels increase from ∼0.8% to ∼1.2%, with a sidelobe distribution that is almost identical. If we instead mask out the labs and road, the sidelobes also increase marginally, to ∼1.3%, but with a slightly different distribution than in the unmasked case. Throwing away the antennas in the grove of trees has by far the most pronounced effect, with sidelobe levels that increase to 2.4% and that are more complexly distributed at the 1–2% level. The net effect of all the masking is a beam that has near-in sidelobes as deep as 3.4%, and with a fairly complex sidelobe distribution.

The trees are by far the most serious constraint on the resulting beam pattern. Since the Reuleaux triangle is well matched to the triangular site geometry, masking the overall site perimeter has a relatively minimal effect. Similarly, one could easily rearrange the antennas masked by the road and the lab buildings to be situated close to these regions so that their effect is minimized. But the grove of trees is a large chunk of masked land that is situated right at a location which eliminates the longest baselines with the northernmost apex of the Reuleaux triangle. Furthermore, since the trees are even taller than the antennas, the shadowing constraints from the trees are exacerbated and the true masked region caused by the trees might effectively be larger in all directions. That is, unlike the situation for the road, one can't simply nestle an antenna right up to the boundary of the masked region to improve the resulting configuration. If anything, the true effect of the trees on the synthesized beam will be worse than what is shown in Figure 5.

A similar experiment with the best of the uniform random distributions, case 9, is shown in Figure 6. In this case, where the sidelobe levels and distribution already were more complex, the trees don't have such a dramatically worse impact on the sidelobe levels than the site perimeter, the road and the labs.

5. Gaussian Distributions with Different FWHM

Since the gaussian distribution appears to yield synthesized beams with smaller and simpler sidelobes than a uniform distribution of antennas, I ran random distributions of gaussians with different FWHM to explore the tradeoff between the size of the synthesized beam and the level of the near-in sidelobes. As in §2, I did not mask out antennas that fell in disallowed regions. I again rotated the Reuleaux triangle by 15° and scaled the configurations to fit the site in the same way as in §2, with minimum separations of 11m between antennas.

The tradeoff between the beam size and near-in sidelobe levels is listed in Table 3. I ran 10 random distributions for each case and selected the configuration with the lowest near-in sidelobes to list in the table. Also included are the results from the best uniform distribution of antennas (§3). Note that a gaussian distribution that yields the same size beam as the best uniform distribution (82″) has sidelobe levels of only 1.7%, compared to the uniform distribution sidelobe levels of 2.7%. This again suggests that it would be advantageous to choose a configuration that includes some tapering of the antenna distribution. Of course, a tapered distribution yields a minimum beam size; here, this minimum beam size is about 82″.

6. Thick Donuts

In the previous section, we saw that a tapered distribution gives significantly better near-in sidelobes than a uniform distribution of the same beam size, down to the minimum beam size of ∼82″ for a filled Reuleaux array at Hat Creek. If we want to get a smaller beam within the same Reuleaux outline, then we can remove some of the antennas from the center of a gaussian filled Reuleaux distribution to get a thick "donut" configuration. This tapered, thick "donut" configuration should then yield even smaller beam sizes, presumably with the gradual tradeoff in sidelobe levels that we have already seen with the filled arrays in Table 3. In fact, there should be a continuum of choices between an optimized filled Reuleaux array and donuts of increasingly thin widths that yield a smooth distribution of the tradeoff between beam sizes and near-in sidelobe levels.

As an example, I ran one case of a thick donut configuration, without considering multiple random distributions or playing with the width of the donut. For this one case, the resulting beam size was 76.7″, which was indeed smaller than the minimum gaussian array beam size of 82″. The maximum near-in sidelobes were 5.5% without masking, and 9.5% with masking. It would be very interesting to run many random distributions with thick donut constraints to see how well one could do with such an array.

7. Comparison of Beams Generated from Different Masked Configurations

So far, I have quantified the size of the naturally weighted beam and the maximum depth of the near-in sidelobes to characterize the quality of the configuration. It is somewhat more difficult to quantify the distribution and shapes of near-in sidelobes. In Figure 7, I present a comparison of masked configurations discussed in this memo. These include Case 4 of the gaussian distribution with FWHM=2 (§2), Case 9 of the uniform distribution (§3), the single thick-donut configuration (§6), and the non-Reuleaux configuration that Douglas showed in his RAL Science Requirements memo.

8. Optimization of Sidelobe Levels

We have already seen that different random distributions of ∼350 antennas give strikingly different near-in beam characteristics, even for regular arrays that do not include masking; the near-in sidelobe levels vary by a factor of nearly three for only 10 different random distributions. On the other hand, the far-out sidelobe levels fall off as N^{-1} independent of the antenna configuration; even in the few cases presented in Figures 2 and 4, we can see that the sidelobe levels outside a few beam diameters from the center are about the same for the different random distributions. This means that there is a strong case for optimizing the choice of configuration subject to the levels of the near-in sidelobes. This is especially true for dealing with the masking caused by the trees and the other site contraints: it is very difficult to guess by eye how to compensate for a large, irregular region like the tree mask.

We have access to Leonia Kogan's optimization program, which does just what we want here. Given an input configuration geometry and a masking file that specifies allowed regions within that geometry, Kogan's program optimizes for minimum sidelobe level over the central 10, 20, or 40 beam diameters. At $\nu = 1.4$ GHz, the primary beam of the 6m dish is 2.4° or about 110 beam diameters if the synthesized beam is 80″. If we run Kogan's code to optimize over 40 beam diameters, this should be plenty large enough to get into the far-out regime where the sidelobe levels have already fallen off as N^{-1}.

9. Conclusions

(1) Different random distributions can give very different near-in sidelobe levels and distributions.

(2) For a given beam size, a tapered or gaussian-like distribution gives better near-in sidelobes than a uniform distribution of antennas.

(3) For tapered Reuleaux configurations, masking for the grove of trees on the southern part of the site introduces much worse near-in sidelobes than does masking for the overall site perimeter, the labs, or the road. Furthermore, given the height of the trees, shadowing constraints may make the effective region for the tree mask even larger than we have already assumed.

(4) A gaussian distribution scaled to the site gives a minimum average beam size of roughly 82″. This is about 20% larger in diameter than the 69″ beam from a uniform array that is not confined to within the Reuleaux outline on the Hat Creek site. It may be worthwhile to run models

of a tapered distribution of antennas within a thick "donut" Reuleaux triangle to see if we can achieve desirable near-in sidelobes with a smaller beam.

(5) Including masking for the site perimeter, labs, and trees, the near-in sidelobes are 3.4% for a gaussian Reuleaux distribution with an 86.5″ beam. The 69″ beam from the masked uniform array that is confined only by the site perimeter has near-in sidelobes of 6.8%.

(6) The sidelobes far out in the beam fall off as N^{-1}, independent of the choice of configuration. It should be worthwhile to optimize the choice of configuration for near-in sidelobes; such an optimization will likely yield the best compensation for the masking due to the trees and other site constraints, whether the configuration is confined to a Reuleaux outline or not. Kogan's algorithm allows for optimization of near-in sidelobes over an area up to 40 beam diameters subject to an input topography mask; this should be overly sufficient for designing the ATA configuration.

Table 1: Random Gaussian Distributions with FWHM = 2: No Masking

Case	$B_{maj}('')$	$B_{min}('')$	$B_{mean}('')$	nants
1	90.0	80.8	85.3	328
2	84.6	81.6	83.1	346
3	86.6	82.0	84.3	340
4	85.9	84.5	85.2	338
5	85.7	83.6	84.6	324
6	85.5	80.0	82.7	315
7	85.1	83.5	84.3	354
8	87.3	78.3	82.7	329
9	84.4	80.6	82.5	333
10	86.0	79.1	82.5	332

Table 2: Random Uniform Distributions: No Masking

Case	$B_{maj}('')$	$B_{min}('')$	$B_{mean}('')$	nants
1	83.3	76.0	79.5	351
2	80.0	75.8	77.9	351
3	81.8	78.8	80.3	365
4	81.5	75.4	78.4	353
5	87.0	72.5	79.4	374
6	82.2	76.4	79.3	344
7	83.4	73.8	78.4	368
8	85.8	73.4	79.3	350
9	84.5	79.1	81.8	367
10	79.8	77.1	78.4	359

Table 3: Empirical Beam Size vs. Near-in Sidelobe Levels: No Masking

Case	B_{mean}	Max Near-in Sidelobe
Gaus FWHM=1	$90.2''$	0.5%
Gaus FWHM=2	85.2	0.8
Gaus FWHM=4[a]	81.9	1.7
Unif Iter 9 (§3)	81.8	2.7

[a]Gaussians of larger FWHM give essentially the same beam size given the choice of scaling to the site.

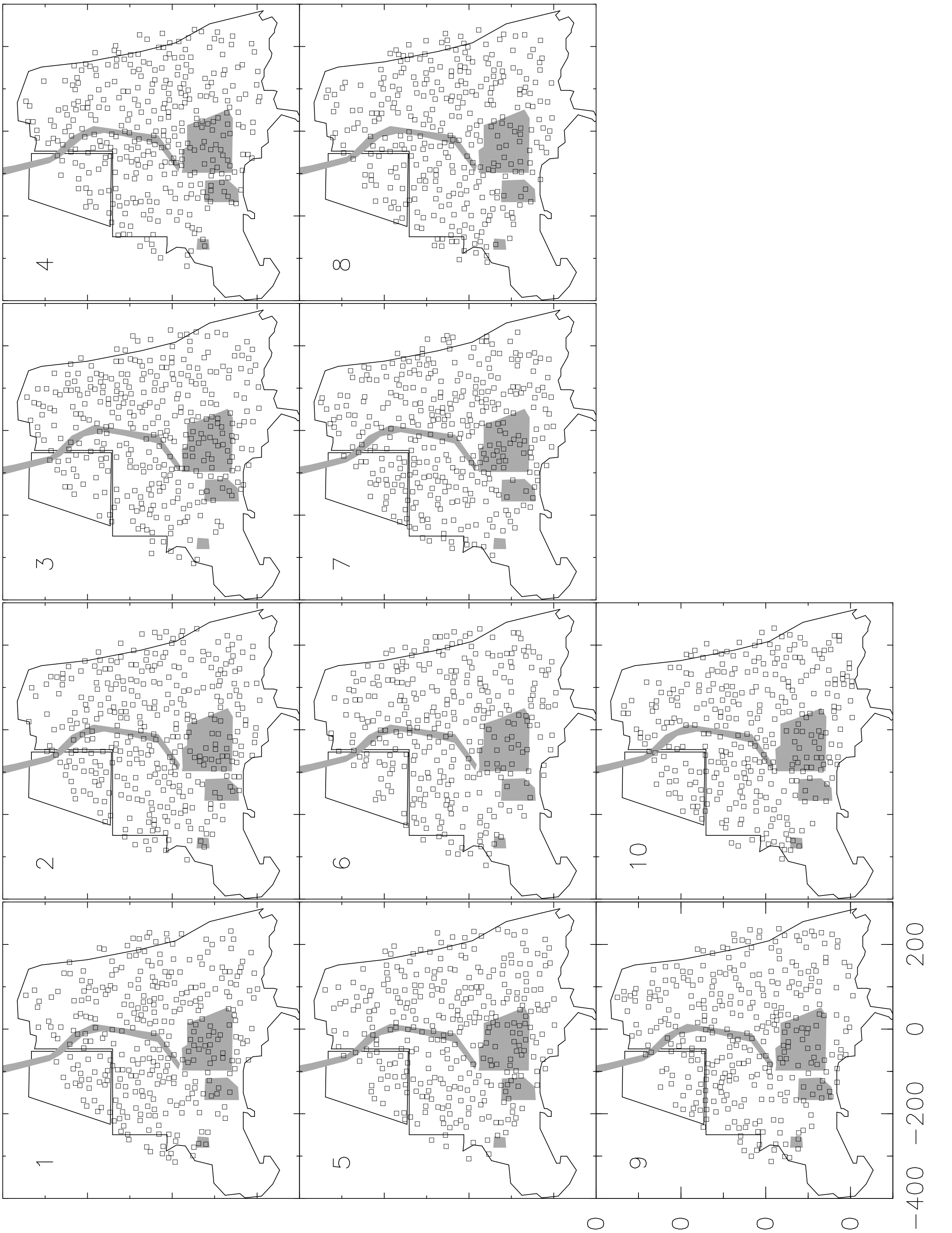

Fig. 1.— Antenna configurations from 10 random gaussian distributions with FWHM=2, constrained to lie within the outline of a Reuleaux triangle. No masking for the site perimeter, labs, road, or grove of trees is included.

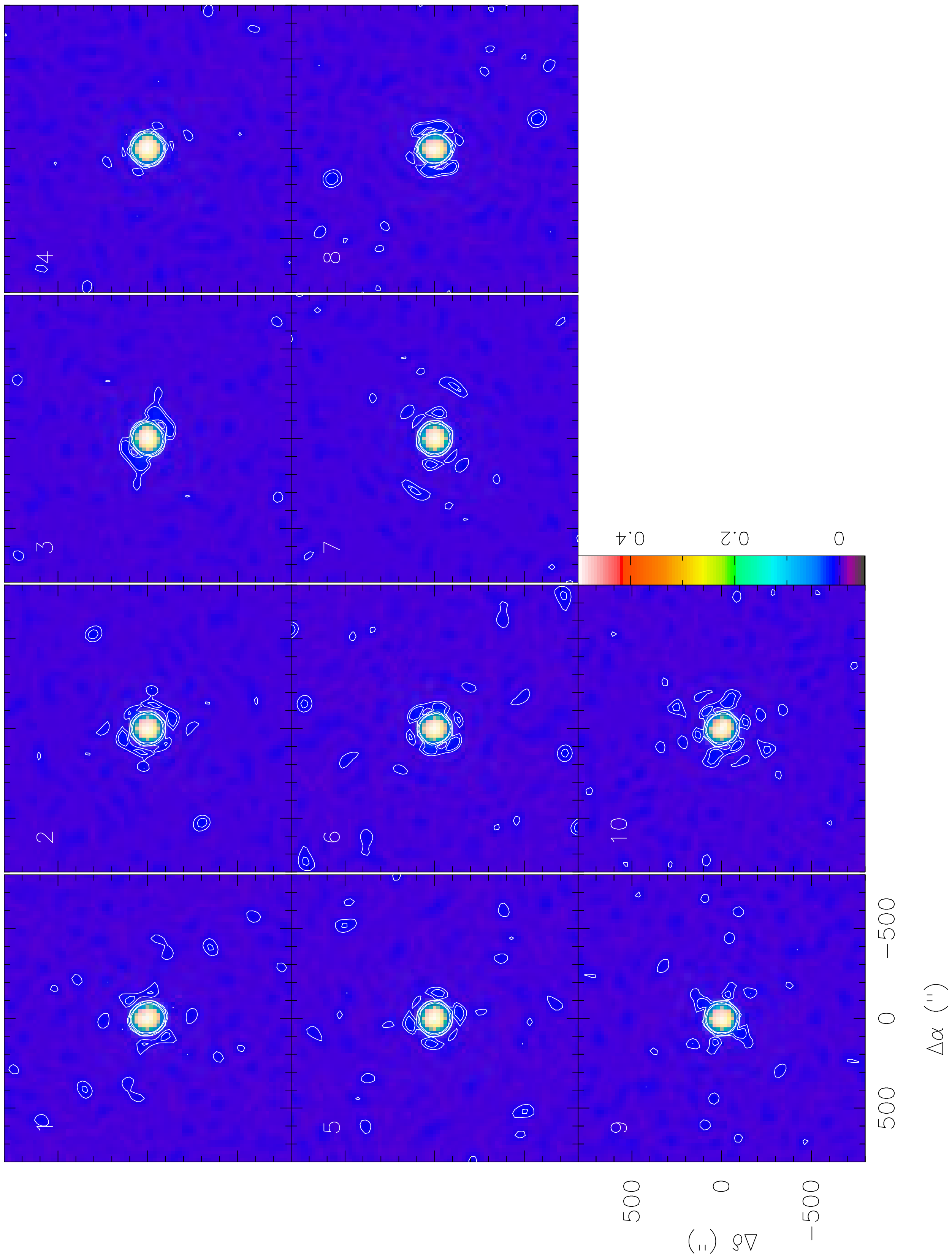

Fig. 2.— Naturally weighted beams from random gaussian distributions shown in Figure 1, with ν = 1.4GHz. No masking is included. Contours are shown at ±.005, ±.01, .03, .05 of the peak.

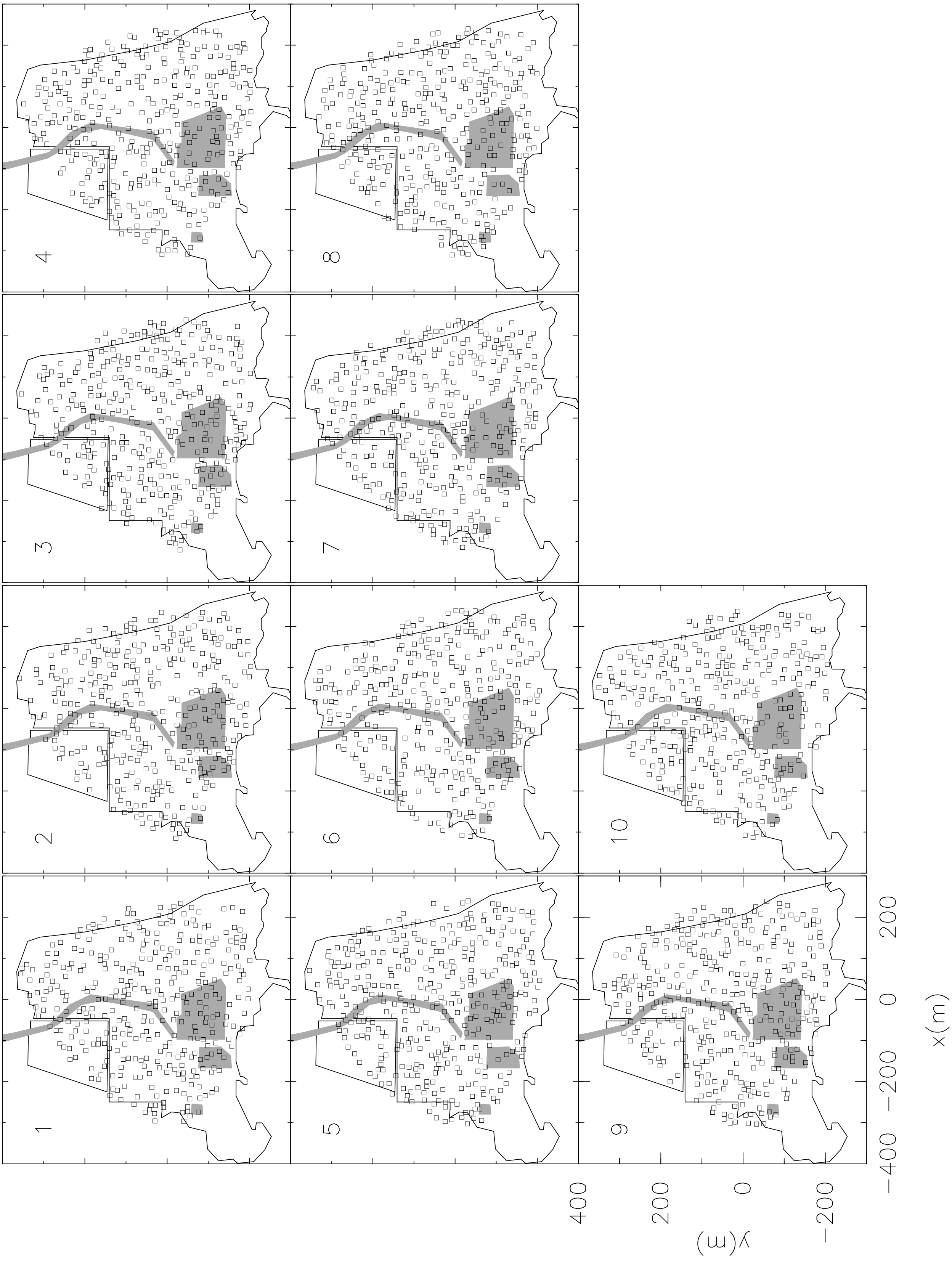

Fig. 3.— Antenna configurations from 10 random uniform distributions, constrained to lie within the outline of a Reuleaux triangle. No masking for the site perimeter, labs, road, or grove of trees is included.

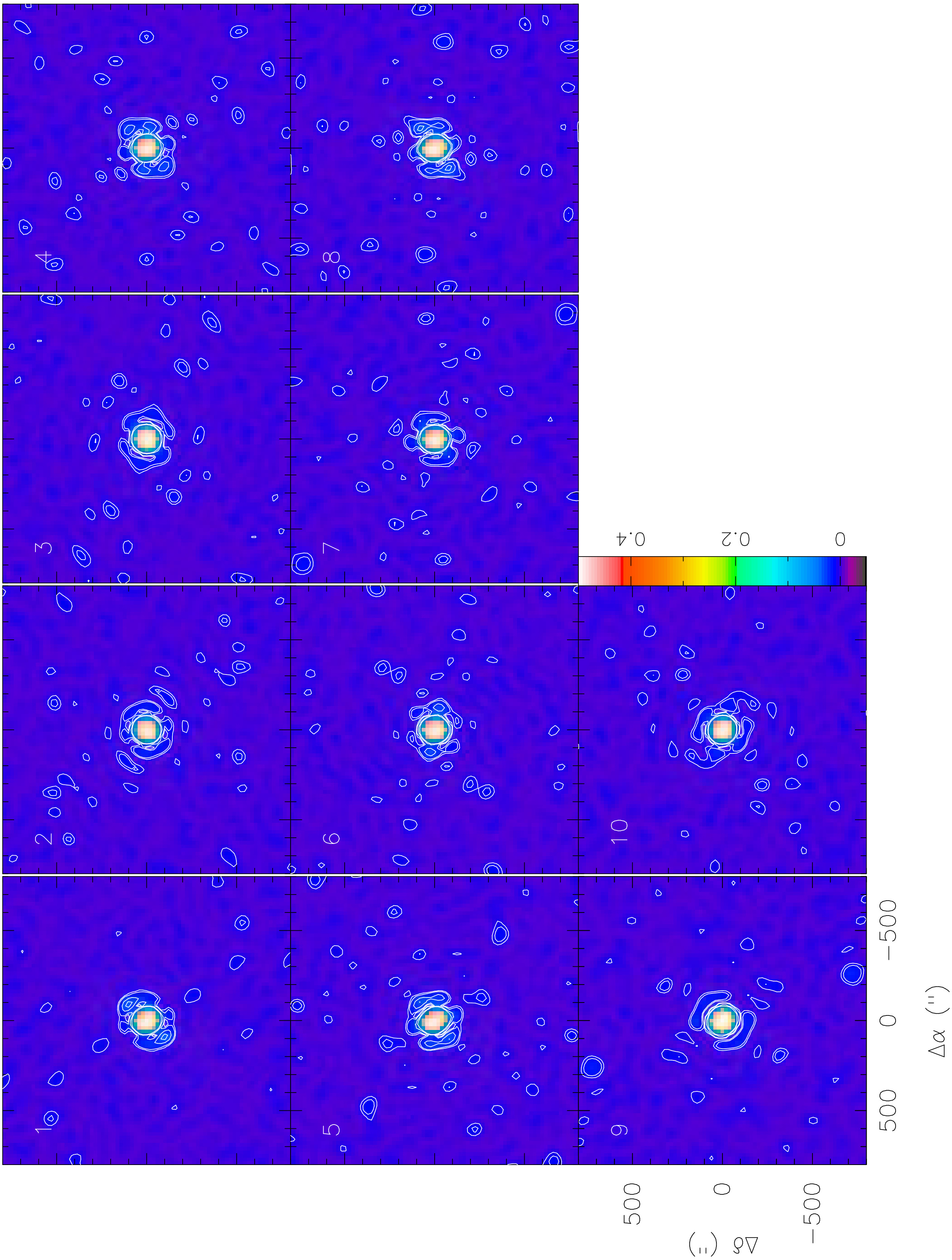

Fig. 4.— Naturally weighted beams from random uniform distributions shown in Figure 3, with ν = 1.4GHz. No masking is included. Contours are shown at $\pm .005$, $\pm .01$, .03, .05 of the peak.

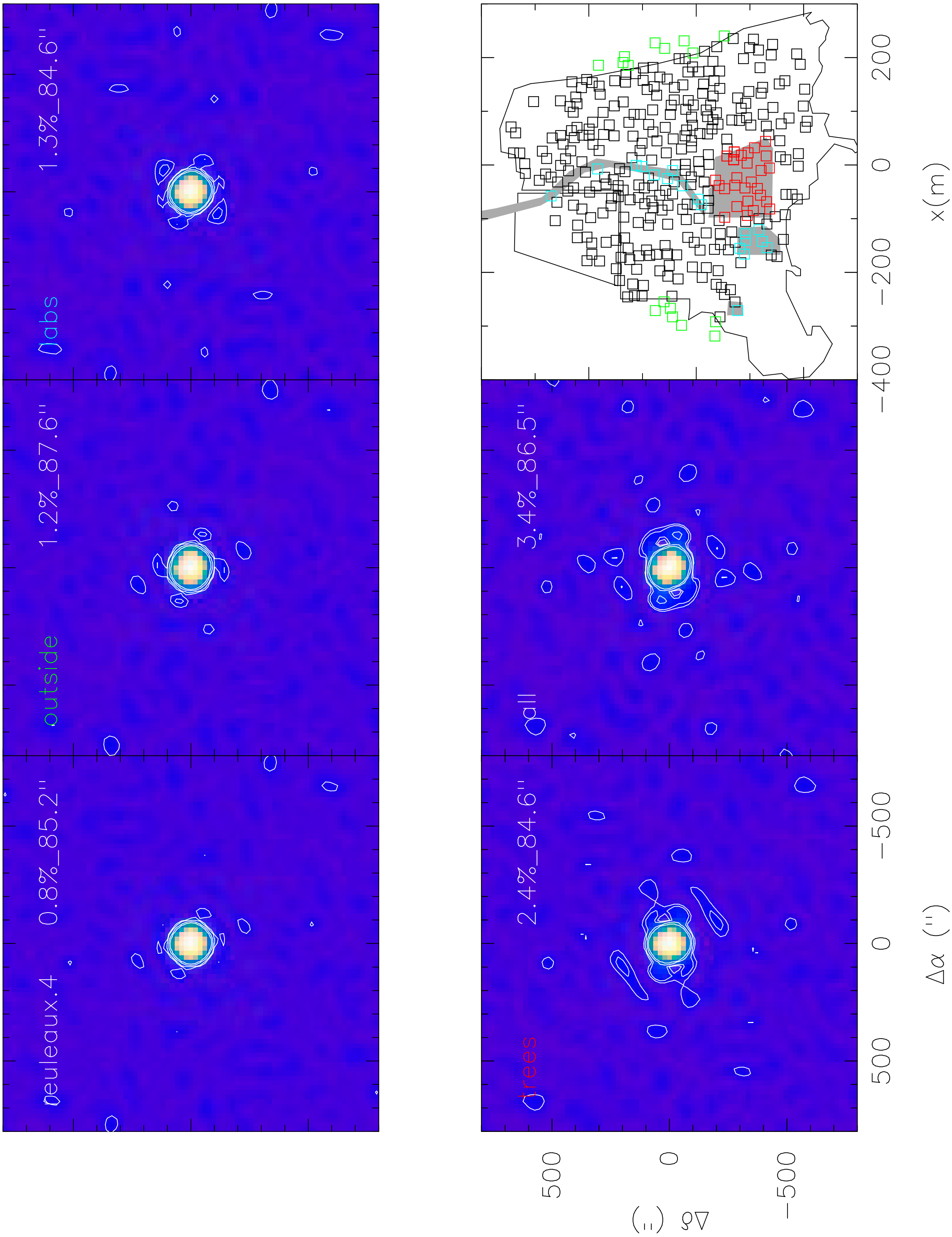

Fig. 5.— Effect of masking on gaussian Case 4 from Figure 1. Synthesized beams after (*upper left*) no masking, (*upper middle*) masking for the perimeter of the allowed site, (*upper right*) the two labs and the road, (*lower left*) the grove of trees, and (*lower middle*) all of the above. Contours are shown at ±.005, ±.01, .03, .05 of the peak. The maximum near-in sidelobe and size of the synthesized beams are shown in the upper right corner of each panel. (*lower right*) The antenna configuration is color-coded to show which antennas are eliminated with each set of masking.

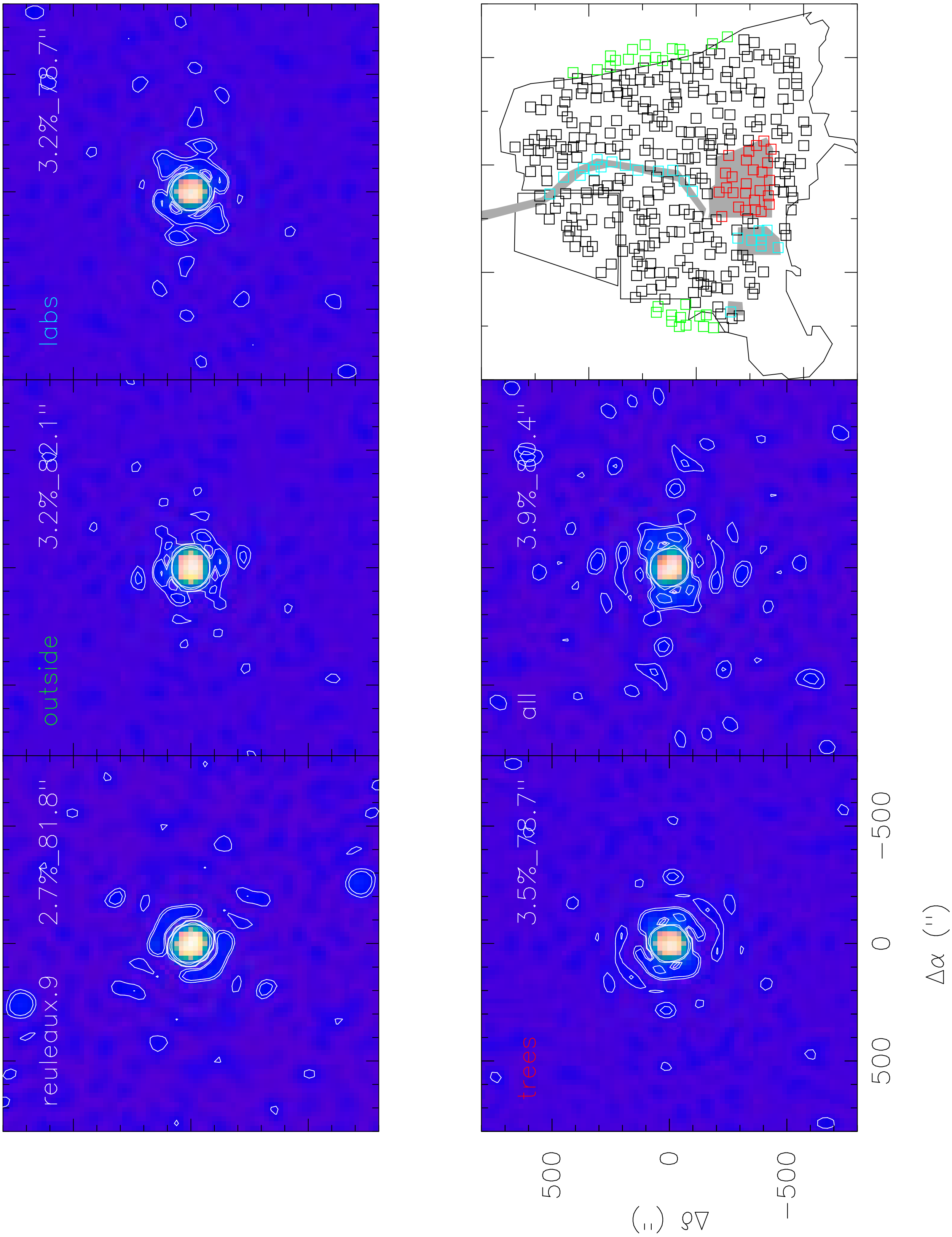

Fig. 6.— Same as in Figure 5, but for the best-case uniform distribution, Case 9 from Figure 3.

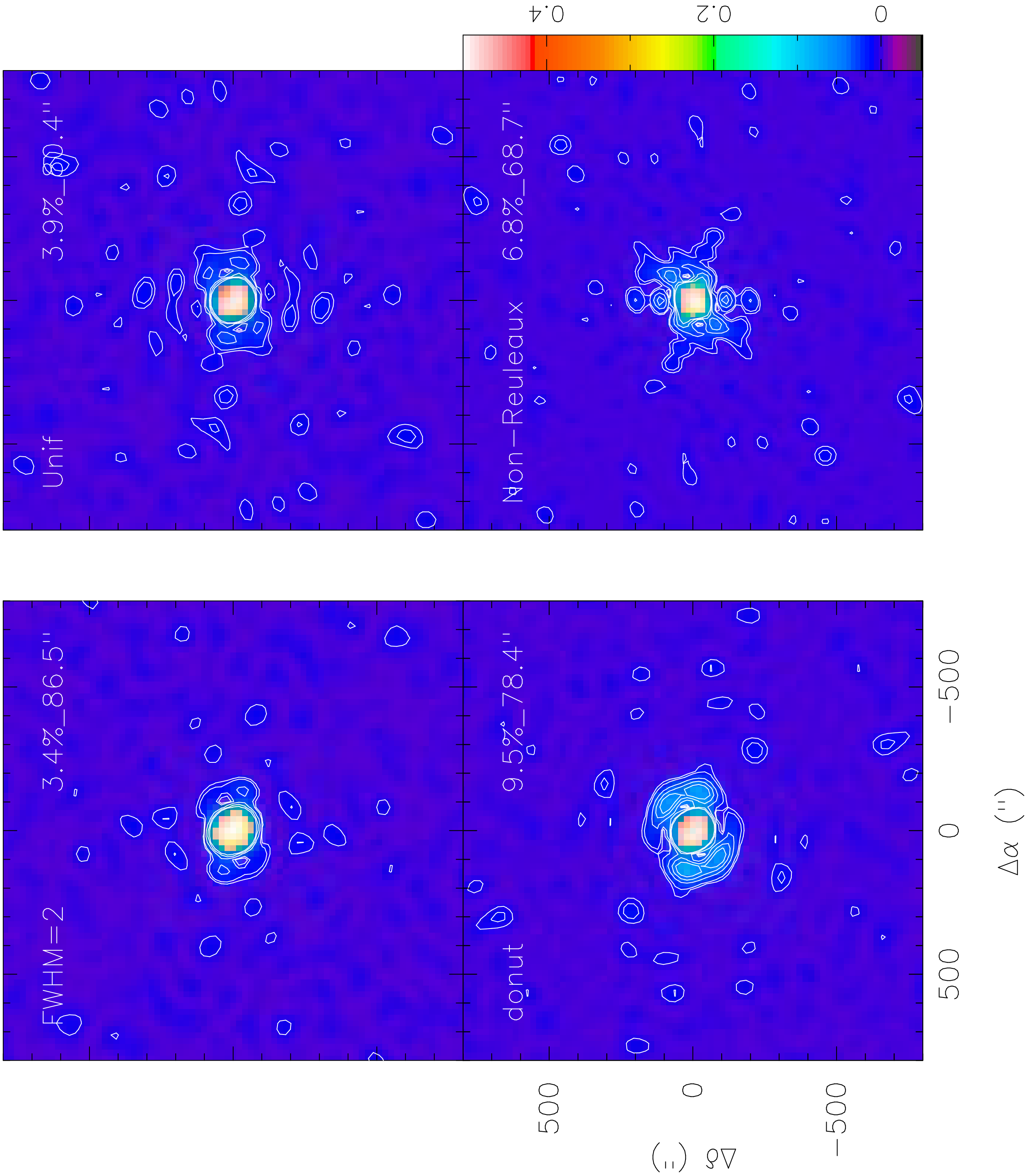

Fig. 7.— Naturally weighted beams from different configurations discussed in this memo (see §7). All configurations now include masking for the site perimeter, labs, road, and trees. Contours are shown at ±.005, ±.01, .03, .05, .07 of the peak. The maximum near-in sidelobe and size of the synthesized beams are shown in the upper right corner of each panel.